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New Hints for Testing Anomalous Top-Quark Interactions at Future Linear Colliders

Bohdan GRZADKOWSKI [1), a)] and Zenrō HIOKI [2), b)]

1) *Institute of Theoretical Physics, Warsaw University*
Hoża 69, PL-00-681 Warsaw, POLAND

2) *Institute of Theoretical Physics, University of Tokushima*
Tokushima 770-8502, JAPAN

ABSTRACT

Angular and energy distributions for leptons and bottom quarks in the process $e^+e^- \to t\bar{t} \to \ell^\pm/\overset{(-)}{b}\cdots$ have been calculated assuming the most general top-quark couplings. The double distributions depend both on modification of the $t\bar{t}$ production and $\overset{(-)}{t} \to \overset{(-)}{b} W$ decay vertices. However, the leptonic angular distribution turned out to be totally insensitive to non-standard parts of Wtb vertex. Distributions of decay products for polarized top quark in its rest frame have been also calculated. It has been found that the factorization of energy and angular dependence for the double leptonic distribution noticed earlier for the Standard Model survives even if one allows for deviations from the V−A interactions, and the SM angular leptonic distribution turned out to be preserved by anomalous decay vertex.

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[a)]E-mail address: bohdan.grzadkowski@fuw.edu.pl
[b)]E-mail address: hioki@ias.tokushima-u.ac.jp

1. Introduction

In spite of the fact that the top quark has been discovered already several years ago [1] its interactions are still unknown. It remains an open question if the top-quark couplings obey the Standard Model (SM) scheme of the electroweak forces or there exists a contribution from physics beyond the SM. In this letter we will try to construct some tools which could help to answer that question at future e^+e^- linear colliders and therefore reveal the structure of fundamental interactions beyond the SM.

The top quark decays immediately after being produced [2] and its huge mass $m_t \simeq 174$ GeV leads to a decay width Γ_t much larger than $\Lambda_{\rm QCD}$. Therefore the decay process is not influenced by fragmentation effects and the decay products will provide useful information on top-quark properties. Here we will consider distributions either of $\ell^\pm$ in the inclusive process $e^+e^- \to t\bar{t} \to \ell^\pm \cdots$ or bottom quarks from $e^+e^- \to t\bar{t} \to \overset{(-)}{b} \cdots$. We are also studying decays of polarized top quark to $\ell^+/b + \cdots$ in its rest frame. It turns out that the analysis of leptonic and b-quark final states are similar and could be presented simultaneously.

2. Framework and Formalism

We will parameterize $t\bar{t}$ couplings to the photon and the Z boson in the following way

$$\Gamma^\mu_{vt\bar{t}} = \frac{g}{2}\,\bar{u}(p_t)\left[\gamma^\mu\{A_v + \delta A_v - (B_v + \delta B_v)\gamma_5\} + \frac{(p_t - p_{\bar{t}})^\mu}{2m_t}(\delta C_v - \delta D_v\gamma_5)\right]v(p_{\bar{t}}), \quad (1)$$

where g denotes the $SU(2)$ gauge coupling constant, $v = \gamma, Z$, and

$$A_\gamma = \frac{4}{3}\sin\theta_W, \quad B_\gamma = 0, \quad A_Z = \frac{1}{2\cos\theta_W}\Big(1 - \frac{8}{3}\sin^2\theta_W\Big), \quad B_Z = \frac{1}{2\cos\theta_W}$$

denote the SM contributions to the vertices. Among the above non-SM form factors, $\delta A_{\gamma,Z}$, $\delta B_{\gamma,Z}$, $\delta C_{\gamma,Z}$ describe CP-conserving while $\delta D_{\gamma,Z}$ parameterizes CP-violating interactions. Similarly, we will adopt the following parameterization of the Wtb vertex suitable for the t and $\bar{t}$ decays:

$$\Gamma^\mu_{Wtb} = -\frac{g}{\sqrt{2}}V_{tb}\,\bar{u}(p_b)\Big[\gamma^\mu(f_1^L P_L + f_1^R P_R) - \frac{i\sigma^{\mu\nu}k_\nu}{M_W}(f_2^L P_L + f_2^R P_R)\Big]u(p_t),$$

$$\bar{\Gamma}^{\mu}_{Wtb} = -\frac{g}{\sqrt{2}} V^{*}_{tb}\, \bar{v}(p_{\bar{t}}) \Big[\gamma^{\mu} (\bar{f}_1^L P_L + \bar{f}_1^R P_R) - \frac{i\sigma^{\mu\nu} k_{\nu}}{M_W} (\bar{f}_2^L P_L + \bar{f}_2^R P_R) \Big] v(p_{\bar{b}}), \quad (2)$$

where $P_{L/R} = (1 \mp \gamma_5)/2$, V_{tb} is the (tb) element of the Kobayashi-Maskawa matrix and k is the momentum of W. On the other hand, it will be assumed here that interactions of leptons with gauge bosons are properly described by the SM. Through the calculations all fermions except the top quark will be considered as massless. We will also neglect terms quadratic in non-standard form factors.

Using the technique developed by Kawasaki, Shirafuji and Tsai [3] one can derive the following formula for the inclusive distributions of the top-quark decay product f in the process $e^+e^- \to t\bar{t} \to f + \cdots$ [4]:

$$\frac{d^3\sigma}{d\boldsymbol{p}_f/(2p_f^0)}(e^+e^- \to f + \cdots) = 4 \int d\Omega_t \frac{d\sigma}{d\Omega_t}(n,0) \frac{1}{\Gamma_t} \frac{d^3\Gamma_f}{d\boldsymbol{p}_f/(2p_f^0)}(t \to f + \cdots), \quad (3)$$

where Γ_t is the total top-quark decay width and $d^3\Gamma_f$ is the differential decay rate for the process considered. $d\sigma(n,0)/d\Omega_t$ is obtained from the angular distribution of $t\bar{t}$ with spins s_+ and s_- in $e^+e^- \to t\bar{t}$, $d\sigma(s_+, s_-)/d\Omega_t$, by the following replacement:

$$s_{+\mu} \to n_{\mu}^f = -\Big[g_{\mu\nu} - \frac{p_{t\mu} p_{t\nu}}{m_t^2} \Big] \frac{\sum \int d\Phi\, \bar{B} \Lambda_+ \gamma_5 \gamma^{\nu} B}{\sum \int d\Phi\, \bar{B} \Lambda_+ B}, \quad s_{-\mu} \to 0, \quad (4)$$

where the matrix element for $t(s_+) \to f + \cdots$ was expressed as $\bar{B} u_t(p_t, s_+)$, $\Lambda_+ \equiv \not{p}_t + m_t$, $d\Phi$ is the relevant final-state phase-space element and $\sum$ denotes the appropriate spin summation.

3. Distributions in e^+e^- CM Frame

In this section we will present results for $d^2\sigma/dx_f d\cos\theta_f$ of the top-quark decay product f, where f could be either $\ell^{\pm}$ or $\overset{(-)}{b}$, x_f denotes the normalized energy of f and θ_f is the angle between the e^- beam direction and the direction of f momentum in the e^+e^- CM frame.

Direct calculations performed in presence of the general decay vertex (2) lead to the following result for the n_{μ}^f vector defined in eq.(4):

$$n_{\mu}^f = \alpha^f \left(g_{\mu\nu} - \frac{p_{t\mu} p_{t\nu}}{m_t^2} \right) \frac{m_t}{p_t p_f} p_f^{\nu} \quad (5)$$

where for a given final state f, α^f is a calculable depolarization factor

$$\alpha^f = \begin{cases} 1 & \text{for } \ f = \ell^+ \\ \\ \dfrac{2r-1}{2r+1}\Big[\,1 + \dfrac{8\sqrt{r}(1-r)}{(2r-1)(2r+1)}\mathrm{Re}(f_2^R)\,\Big] & \text{for } \ f = b \end{cases} \tag{6}$$

with $r \equiv (M_W/m_t)^2$.

It should be emphasized here that the above result means that there are no corrections to the "polarization vector" n_μ^ℓ for the semileptonic top-quark decay. As it will be shown in the next section, that has important consequences for leptonic distributions for polarized top quark in its rest frame. On the other hand, one can see that the corrections to α^b could be substantial as the kinematical suppression factor in the leading term $2r-1 (= -0.56)$ could be canceled by the appropriate contribution from the non-standard form factor f_2^R.

Applying the strategy described above and adopting the general formula for the $t\bar{t}$ distribution $d\sigma(s_+, s_-)/d\Omega_t$ from ref.[5], one obtains the following result for the double distribution of the angle and the rescaled energy of f:

$$\frac{d^2\sigma}{dx_f d\cos\theta_f} = \frac{3\pi\beta\alpha_{\mathrm{EM}}^2}{2s} B_f \Big[\,\Theta_0^f(x_f) + \cos\theta_f\,\Theta_1^f(x_f) + \cos^2\theta_f\,\Theta_2^f(x_f)\,\Big], \tag{7}$$

where β is the top velocity, α_{EM} is the fine structure constant and B_f denotes the appropriate branching fraction. The energy dependence is specified by the functions $\Theta_i^f(x_f)$, explicit forms of which are shown in Appendix. They are parameterized both by production and decay form factors.

The angular distribution[♯1] for f could be easy obtained[♯2] from eq.(7) by the integration over the energy of f:

$$\frac{d\sigma}{d\cos\theta_f} \equiv \int_{x_-}^{x_+} \frac{d^2\sigma}{dx_f d\cos\theta_f} dx_f = \frac{3\pi\beta\alpha_{\mathrm{EM}}^2}{2s} B_f \left(\Omega_0^f + \Omega_1^f\cos\theta_f + \Omega_2^f\cos^2\theta_f\right), \tag{8}$$

[♯1] The energy distributions could be, of course, obtained through the integration of eq.(7) over $\cos_{\theta_f}$. Results for the lepton-energy spectrum calculated for the general form factors considered here could be found in ref.[5], while the energy spectrum for b will be published elsewhere [6].

[♯2] In the SM limit we do reproduce results obtained earlier by Arens and Sehgal [7]. The *CP*-violating contributions for the semileptonic decays have been compared with the results found by Poulose and Rindani. After correcting several misprints in their formula (9) in ref.[8] our results agreed. In ref.[9] certain observables depending on *CP* violation in the production and the decay processes have been discussed in the framework of the two-Higgs doublet and supersymmetric extensions of the SM. Implicitly, expectation values for the observables considered there were also sensitive to lepton and b-quark angular distributions.

where $\Omega_i^f = \int_{x_-}^{x_+} \Theta_i^f dx$ and $x_\pm$ define kinematical energy range. The decay vertex is entering our double distribution, eq.(7), i) through the functions $F^f(x_f)$, $G^f(x_f)$ and $H_{1,2}^f(x_f)$ defined in Appendix, and ii) through the depolarization factor α^f. All the non-SM parts of F^f, G^f and $H_{1,2}^f$ disappear upon integration over the energy x_f both for ℓ^+ and b, as it could be seen from the explicit forms for Ω_i^f given in Appendix. As $\alpha^f = 1$ for the leptonic distribution, we conclude that the whole dependence of the lepton distribution on non-standard structure of the top-quark decay vertex drops out through the integration over the energy! However, one can expect substantial modifications for the bottom-quark distribution since corrections to α^b could be large.

The fact that the angular leptonic distribution is insensitive to corrections to the V−A structure of the decay vertex allows for much more clear tests of the production vertices through measurement of the distribution, since that way we can avoid a contamination from non-standard structure of the decay vertex. As an illustration we define a *CP*-violating asymmetry which could be constructed using the angular distributions of f and $\bar{f}$:

$$\mathcal{A}_{CP}(\theta_f) = \Big[\frac{d\sigma^+(\theta_f)}{d\cos\theta_f} - \frac{d\sigma^-(\pi-\theta_f)}{d\cos\theta_f}\Big] \Big/ \Big[\frac{d\sigma^+(\theta_f)}{d\cos\theta_f} + \frac{d\sigma^-(\pi-\theta_f)}{d\cos\theta_f}\Big], \tag{9}$$

where $d\sigma^{+/-}$ is referring to f and $\bar{f}$ distributions, respectively. Since $\theta_f \to \pi - \theta_{\bar{f}}$ under *CP*, the asymmetry defined above is a true measure of *CP* violation.[♯3] It is straightforward to find that the denominator in eq.(9) is

$$\frac{d\sigma^+(\theta_f)}{d\cos\theta_f} + \frac{d\sigma^-(\pi-\theta_f)}{d\cos\theta_f} = 2\Big[\frac{d\sigma^+(\theta_f)}{d\cos\theta_f}\Big]^{(0)}, \tag{10}$$

where the subscript (0) denotes the SM contribution to eq.(8), while the numerator becomes

$$\frac{d\sigma^+(\theta_f)}{d\cos\theta_f} - \frac{d\sigma^-(\pi-\theta_f)}{d\cos\theta_f} = \frac{3\pi\beta\alpha_{\rm EM}^2}{2s} B_f$$
$$\times 2\left[\, \alpha_0^f\Big(1 - \frac{1-\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta}\Big)\Big[\, (1-3\cos^2\theta_f){\rm Re}(F_1) - 2\cos\theta_f{\rm Re}(F_4)\,\Big]\right.$$

[♯3] Angular asymmetries have been also discussed in refs.[8] and [10].

$$
-\alpha_1^f(1-\beta^2)\Big\{\Big(1-\frac{1}{2\beta}\ln\frac{1+\beta}{1-\beta}\Big)\mathrm{Re}(D_{VA}^{(0)})(1-3\cos^2\theta_f)
$$
$$
-\Big[\,E_A^{(0)}-(E_V^{(0)}+E_A^{(0)})\frac{1}{2\beta}\ln\frac{1+\beta}{1-\beta}\,\Big]\cos\theta_f\,\Big\}\mathrm{Re}(f_2^R-\bar{f}_2^L)\Bigg], \qquad (11)
$$

for the coefficients $F_{1,4}$ specified in Appendix, and we expressed α^f as $\alpha_0^f+\alpha_1^f\mathrm{Re}(f_2^R)$ with

$$
\begin{array}{lll}
\alpha_0^f=1, & \alpha_1^f=0 & (\text{for } f=\ell), \\
\alpha_0^f=\dfrac{2r-1}{2r+1}, & \alpha_1^f=\dfrac{8\sqrt{r}(1-r)}{(1+2r)^2} & (\text{for } f=b),
\end{array}
$$

up to linear terms in the non-SM parameters.

As one could have anticipated, the asymmetry for $f=\ell$ is sensitive to CP violation originating exclusively from the production mechanism, i.e. it depends only on $F_{1,4}$ that contain contributions from CP-violating form factors δD_γ and δD_Z while the decay vertex enters with the SM CP-conserving coupling. For bottom quarks the effect of the modification of the decay vertex is contained in corrections to b and $\bar{b}$ depolarization factors, $\alpha^b+\alpha^{\bar{b}}=\alpha_1^b\mathrm{Re}(f_2^R-\bar{f}_2^L)$ with SM CP-conserving contribution from the production process.[♯4] As it is seen from fig.1 for $\sqrt{s}=1$ TeV the asymmetry could be quite large, e.g., reaching for the semileptonic decays $\sim 20\%$ for $\mathrm{Re}(\delta D_\gamma)=\mathrm{Re}(\delta D_Z)=0.2$.

CP-violating form factors discussed here could be also generated within the SM. However, it is easy to notice that first non-zero contribution to $\delta D_{\gamma,Z}$ would require at least two loops. For the top-quark decay process CP violation could appear at the one-loop level, however it is strongly suppressed by double GIM mechanism [12]. Therefore we can conclude that experimental detection of CP-violating form factors considered here would be a clear indication for physics beyond the SM.

[♯4] One can show that $f_1^{L,R}=\pm\bar{f}_1^{L,R}$ and $f_2^{L,R}=\pm\bar{f}_2^{R,L}$ where upper (lower) signs are those for CP-conserving (-violating) contributions [11]. Therefore any CP-violating observable defined for the top-quark decay must be proportional to $f_1^{L,R}-\bar{f}_1^{L,R}$ or $f_2^{L,R}-\bar{f}_2^{R,L}$.

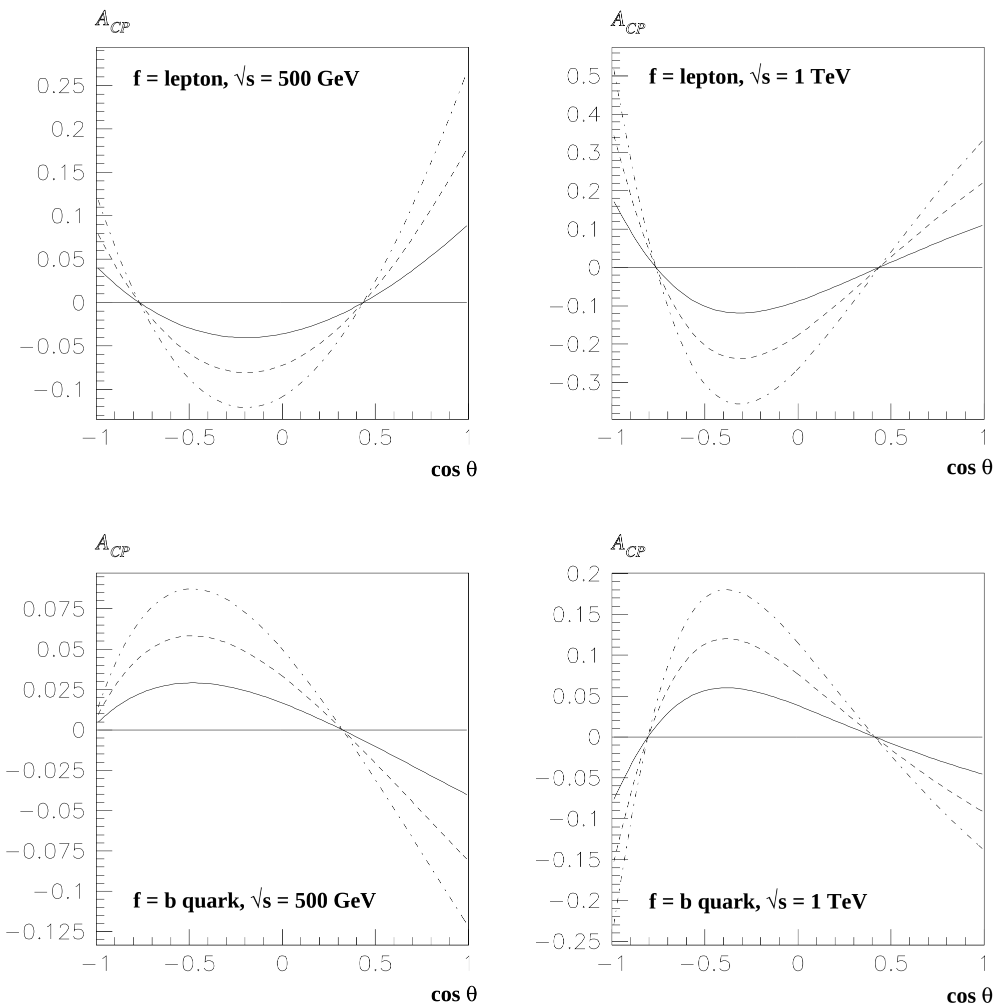

Figure 1: The CP-violating asymmetry $\mathcal{A}_{CP}(\theta_f)$ defined in eq.(9) as a function of $\cos\theta_f$ for leptonic and b-quark distributions for $\mathrm{Re}(\delta D_\gamma) = \mathrm{Re}(\delta D_Z) = \mathrm{Re}(f_2^R - \bar{f}_2^L)$ =0.1 (solid line), 0.2 (dashed line), 0.3 (dash-dotted line) at $\sqrt{s} = 500$ GeV and 1 TeV collider energy.

4. Distributions in Top-Quark Rest Frame

It is instructive to consider decays of a polarized top quark in its rest frame in presence of the general decay vertex defined by eq.(2). It turns out that the leptonic angular and energy distribution has a very similar structure to the distribution found [13] for the pure V−A coupling:

$$\frac{1}{\Gamma_\ell}\frac{d^2\Gamma_\ell}{dx^\star\, d\cos\theta_\ell^\star} = \frac{6}{W}x^\star(1-x^\star)\left[1+2\mathrm{Re}(f_2^R)\sqrt{r}\left(\frac{1}{x^\star}-\frac{3}{1+2r}\right)\right]\frac{1+\cos\theta_\ell^\star}{2}, \quad (12)$$

where $W \equiv (1-r)^2(1+2r)$, $x^\star \equiv 2E_\ell/m_t$ is the normalized ℓ^+ energy and $\theta^\star_\ell$ denotes the angle between the top-quark spin and ℓ^+ momentum.

The above formula proves that the factorization of energy and angular dependence in the top-quark rest frame noticed by Jeżabek and Kühn for standard V$-$A top-decay vertex [13] is actually much more general and survives even if the decay vertex given by eq.(2) is considered.[♯5] One should remember that the assumptions adopted here are *i)* $m_b = 0$ and *ii)* neglecting all contributions quadratic in non-standard form factors. Under those assumptions we have proved the factorization of energy and angular dependence.

For the angular distribution one can present results for both ℓ^+ and b in one formula, namely one gets

$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta^\star_f} = \frac{1}{2}[\,1 + \alpha^f \cos\theta^\star_f\,]. \tag{13}$$

The coefficient α^f which measures the amount of information on top-quark spin direction which is being transferred to f direction is exactly the same as the depolarization factor that appeared in eq.(5) in the construction of the top-quark "polarization" vector n^f_μ. We also observe that the angular distribution (13) for ℓ^+ is exactly the same as for the pure V$-$A decay vertex while the one for b receives potentially large corrections from non-standard form factor f^R_2. In table 1 we show the coefficients α^b calculated for various $\mathrm{Re}(f^R_2)$, where one can see the corrections to the SM value of the depolarization factor reach $\sim 50\%$ even for moderate strength of the non-standard contribution to the decay vertex, such as $\mathrm{Re}(f^R_2) = 0.1$.

In the previous section we have noticed that the leptonic angular distribution in e^+e^- CM was not sensitive to modifications of the SM structure for the decay vertex. Indeed, we have observed even though that the anomalous decay vertex was influencing the distribution through functions $F^f(x_f)$, $G^f(x_f)$ and $H^f_{1,2}(x_f)$ however that dependence disappeared after integration over energy x_f both for leptons and

[♯5] It has been found in ref.[14] that the factorization property is approximately preserved by QCD corrections. The formula (12) derived here shows that virtual one-loop QCD corrections precisely preserve the factorization as they just generate contributions to the form factor f^R_2.

$\mathrm{Re}(f_2^R)$	-0.3	-0.2	-0.1	0.0	0.1	0.2	0.3
α^b	-0.84	-0.70	-0.55	-0.41	-0.27	-0.12	$+0.02$

Table 1: The depolarization factor α^b calculated for indicated strengths of the non-standard decay form factor $\mathrm{Re}(f_2^R)$.

bottom quarks. The other source of information on the decay was the depolarization factor α^f, which was however not modified by non-standard interactions in the case of semileptonic decays. As we have just seen, because of that, the angular distribution in the top-quark rest frame was also the same as in the SM. Therefore we can conclude that the independence of the leptonic angular distribution in e^+e^- CM frame to corrections to the decay vertex is equivalent to the preservation of the V$-$A form of the leptonic angular distribution for the polarized top quark in its rest frame. The distribution (13) tells us that the most likely direction of ℓ^+ is the direction pointed by the top-quark spin. Since top-quark spin is determined by the production process and the rest-frame angular distribution is unchanged, the leptonic angular distribution should not be sensitive to modifications of the decay vertex. Our direct calculation confirmed that intuition.

5. Summary and Comments

We have calculated here the angular and energy distributions both for $\overset{(-)}{f}$ in the process $e^+e^- \to t\bar{t} \to \overset{(-)}{f} \cdots$, where $f = \ell$ or b quark, assuming the most general (CP-violating *and* CP-conserving) couplings for $\gamma t\bar{t}$, $Zt\bar{t}$ and Wtb. The bottom-quark mass has been neglected and we have kept only terms linear in modification of the SM vertices. We have found that the double angular and energy distributions depend both on modification of the $t\bar{t}$ production vertices as well as on deviations from the SM at the top-quark decay vertex.

However, the angular distribution for leptons turned out to be absolutely insensitive to variations of the standard V$-$A structure of the Wtb coupling. Therefore the distribution seems to be an excellent tool to measure deviations from the SM in the production process since the experimental results would not be contami-

nated by unknown structure of the decay vertex. In contrast, the bottom-quark angular distribution turned out to be influenced by non-standard corrections to the top-quark decay vertex only through corrections to the depolarization factor.

In order to show some CP violating observable, we have proposed an angular asymmetry which is sensitive to CP-violation in the production of $t\bar{t}$ (for $f = \ell$) and also depends on the CP violation parameters in top-quark decays (for $f = b$). For $\sqrt{s} = 1$ TeV colliders the asymmetry for semileptonic decays could be substantial, e.g., reaching $\sim 20\%$ for CP-violating production form factors of the order of 0.2.

We have also calculated distributions of decay products for polarized top quark in its rest frame for the same most general decay vertex. It has been found that the factorization of energy and angular dependence for the double distribution noticed earlier by Jeżabek and Kühn [13] survives even if one allows for deviations from the V$-$A vertex. Since the lepton angular distribution in the rest frame turned out to be preserved by anomalous parts of the decay vertex, therefore our results for the angular distribution in e^+e^- CM frame could be understood qualitatively.

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Appendix

Here we present explicit formulas for functions Θ_i^f describing energy dependent

coefficients for the angular and energy distributions in eq.(7):

$$
\begin{aligned}
\Theta_0^f(x) \;=\;& \Big[\,\frac{1}{2}(3-\beta^2)D_V - \frac{1}{2}(1-3\beta^2)D_A - (1+\beta^2)\mathrm{Re}(G_1) \\
& -\alpha^f[\,(1-\beta^2)\mathrm{Re}(D_{VA}) - \mathrm{Re}(F_1) + (2-\beta^2)\mathrm{Re}(G_3)\,]\,\Big]\, F^f(x) \\
& +\alpha^f\mathrm{Re}(2D_{VA} - F_1 - G_3)\, G^f(x) \\
& +\Big[\, D_V + D_A + 2\,\mathrm{Re}(G_1) + \alpha^f\mathrm{Re}(2D_{VA} - F_1 + 3G_3)\,\Big]\, H_1^f(x) \\
& -\frac{1}{2}\Big[\, D_V + D_A + 2\,\mathrm{Re}(G_1) + 2\alpha^f\,\mathrm{Re}(D_{VA} + G_3)\,\Big]\, H_2^f(x), \\
\Theta_1^f(x) \;=\;& 2\Big[\,2\mathrm{Re}(E_{VA}) + \alpha^f[\,(1-\beta^2)E_A - \mathrm{Re}(F_4 - G_2)\,]\,\Big] F^f(x) \\
& +2\alpha^f\,\Big[\, E_V + E_A - \mathrm{Re}(F_4 - G_2)\,\Big]\, G^f(x) \\
& -2\,\Big[\,2\mathrm{Re}(E_{VA}) + \alpha^f[\, E_V + E_A - \mathrm{Re}(F_4 - G_2)\,]\,\Big]\, H_1^f(x), \\
\Theta_2^f(x) \;=\;& \Big[\,\frac{1}{2}(3-\beta^2)(D_V + D_A) + (3-\beta^2)\mathrm{Re}(G_1) \\
& +3\alpha^f[\,(1-\beta^2)\mathrm{Re}(D_{VA}) - \mathrm{Re}(F_1) + (2-\beta^2)\mathrm{Re}(G_3)\,]\,\Big]\, F^f(x) \\
& +\alpha^f\mathrm{Re}(2D_{VA} - F_1 + 3G_3)\, G^f(x) \\
& -3\Big[\, D_V + D_A + 2\,\mathrm{Re}(G_1) + \alpha^f\mathrm{Re}(2D_{VA} - F_1 + 3G_3)\,\Big]\, H_1^f(x) \\
& +\frac{3}{2}\Big[\, D_V + D_A + 2\,\mathrm{Re}(G_1) + 2\alpha^f\,\mathrm{Re}(D_{VA} + G_3)\,\Big]\, H_2^f(x),
\end{aligned}
$$

where

$$
\begin{aligned}
& F^f(x) \equiv \frac{1}{B_f}\int d\omega \frac{1}{\Gamma_t}\frac{d^2\Gamma_f}{dx d\omega}, \quad G^f(x) \equiv \frac{1}{B_f}\int d\omega \Big[\,1 - x\frac{1+\beta}{1-\omega}\Big]\frac{1}{\Gamma_t}\frac{d^2\Gamma_f}{dx d\omega}, \\
& H_1^f(x) \equiv \frac{1}{B_f}\frac{1-\beta}{x}\int d\omega (1-\omega)\frac{1}{\Gamma_t}\frac{d^2\Gamma_f}{dx d\omega}, \\
& H_2^f(x) \equiv \frac{1}{B_f}\Big(\frac{1-\beta}{x}\Big)^2\int d\omega (1-\omega)^2\frac{1}{\Gamma_t}\frac{d^2\Gamma_f}{dx d\omega},
\end{aligned}
$$

and ω is defined as $\omega \equiv (p_t - p_f)^2/m_t^2$. The coefficients D_V, D_A, D_{VA}, E_V, E_A, E_{VA}, F_i and G_i could be expressed through the production form factors specified in eq.(1), and the explicit forms of all those relations are available in ref.[5]. The differential top-quark decay rates in e^+e^- CM frame, which appear in the above

definitions of $F^f(x)$, $G^f(x)$ and $H_i^f(x)$ are the following

$$\frac{1}{\Gamma_t}\frac{d^2\Gamma_f}{dxd\omega} = \begin{cases} \dfrac{1+\beta}{\beta}\,\dfrac{3B_\ell}{W}\omega\left[\,1+2\mathrm{Re}(f_2^R)\sqrt{r}\left(\dfrac{1}{1-\omega}-\dfrac{3}{1+2r}\right)\right] & \text{for } f=\ell^+, \\[3ex] \dfrac{1+\beta}{2\beta(1-r)}\delta(\omega-r) & \text{for } f=b. \end{cases}$$

The functions appropriate for $\bar{f}$ could be obtained from the above formulas by changing $F_{1,4}\to -F_{1,4}$, $f_2^R\to \bar{f}_2^L$ and switching sign in front of $\cos\theta_f$ in eq.(7).

Integrals of $\Theta_i^f(x)$ denoted in the main text by Ω_i^f are the following:

$$\begin{aligned}
\Omega_0^f \;&=\; D_V-(1-2\beta^2)D_A-2\,\mathrm{Re}(G_1) \\
&\quad -\alpha^f[\,2(1-\beta^2)\mathrm{Re}(D_{VA})-\mathrm{Re}(F_1)+(3-2\beta^2)\mathrm{Re}(G_3)\,] \\
&\quad +\Big[\,D_V+D_A+2\,\mathrm{Re}(G_1)+\alpha^f\mathrm{Re}(2D_{VA}-F_1+3G_3)\,\Big]\frac{1-\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta}, \\
\Omega_1^f \;&=\; 4\,\mathrm{Re}(E_{VA})+2\alpha^f[\,(1-\beta^2)E_A-\mathrm{Re}(F_4-G_2)\,] \\
&\quad -\{2\mathrm{Re}(E_{VA})+\alpha^f[\,E_V+E_A-\mathrm{Re}(F_4-G_2)\,]\,\}\frac{1-\beta^2}{\beta}\ln\frac{1+\beta}{1-\beta}, \\
\Omega_2^f \;&=\; (3-2\beta^2)[\,D_V+D_A+2\mathrm{Re}(G_1)\,] \\
&\quad +3\alpha^f[\,2(1-\beta^2)\mathrm{Re}(D_{VA})-\mathrm{Re}(F_1)+(3-2\beta^2)\mathrm{Re}(G_3)\,] \\
&\quad -3\Big[\,D_V+D_A+2\,\mathrm{Re}(G_1)+\alpha^f\mathrm{Re}(2D_{VA}-F_1+3G_3)\,\Big]\frac{1-\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta}.
\end{aligned}$$

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